EQUILIBRIUM OF PARTICLES

PROBLEMS
1. The 40-kg block rests on the rough surface. Length of the spring is 180 mm in the position shown. Unstretched length of the spring is 200 mm. Determine the coefficient of friction required for the equilibrium.
2. 4-kg sphere rests on the smooth parabolic surface. Determine the reaction of the surface on the sphere and the mass \( m_B \) required to hold it in the equilibrium position.
3. The hoist shown is used to position heavy workpieces in a lathe. If the cable between pulleys A and B can support a maximum force of 1.2 kN, all other cables can support a maximum force of 2.2 kN, and bar CE can support a maximum compressive force of 2.6 kN, determine the largest weight $W$ that may be lifted.

Note that bar CE is a two force member which can support a force along its longitudinal axis.
4. The pail and its contents have a mass of 10 kg. If the length of cable ABC is 7 m, determine the horizontal distance $x$ for equilibrium. Also find the tension in the cable. The mass of the small pulley at B is small and can be neglected.
5. The cylinder of mass 1 kg having a very small diameter is held against a semi-cylinder with a much larger diameter by two identical springs, which are fixed to points C and C' on the ground. The springs are unstretched when at point A. Knowing that the small cylinder is in equilibrium at point B, what is the spring constant?
6. A small peg $P$ rests on a spring that is contained inside the smooth pipe. The spring exerts an upward force of 284 N on the peg. Determine the point of attachment $A (x, y, 0)$ of cord $PA$ so that the tension in cords $PB$ and $PC$ equals 130 N and 84 N, respectively.
7. The shear leg derrick is used to haul the 200 kg net of fish onto the dock. Determine the compressive force along each of the legs $AB$ and $CB$ and the tension in the winch cable $DB$. Assume the force in each leg acts along its axis.
8. Crate A weighing 580 N rests on the inclined surface by the cable $AB$ and force $P$ which is parallel to the $z$-axis. Determine the tension in the cable $AB$ and force $P$ for equilibrium. Since the crate is mounted on casters, the force exerted by the incline on the crate is perpendicular to the incline.
9. If $W_A = W_B = 1400$ N, determine the force $P$, $T_{AB}$ and the reactions between the collars and bars.
10. The 100-kg collar $A$ rests on the smooth straight fixed bar $CD$ by the cable $AB$. Determine the tension in the cable and the reaction between the collar and bar $CD$. 
11. Collar $A$ weighing 250 N, which can slide freely on the parabolic rod in $xy$ plane, is held in equilibrium by the spring $AB$ and force $P$ which is parallel to $x$-axis. The spring has a constant of 300 N/m and its unstretched length is 0.1 m. Determine the reaction between the collar and rod. Neglect friction.
12. Two bodies weighing 150 N and 200 N, respectively, rest on a cylinder and are connected by a rope as shown. Find the reactions of the cylinder on the bodies, the tension in the rope and the angle $\theta$. Assume all surfaces to be smooth.
SOLUTION 12

FBD of Particle 1

\[ \sum F_x = 0 \]

\[ N_1 \sin \theta - T \cos \theta = 0 \quad \Rightarrow \quad N_1 = \frac{T \cos \theta}{\sin \theta} \]

\[ \sum F_y = 0 \]

\[ N_1 \cos \theta + T \sin \theta - 200 = 0 \]

\[ \left( \frac{T \cos \theta}{\sin \theta} \right) \cos \theta + T \sin \theta - 200 = 0 \]

\[ T \cos^2 \theta + T \sin^2 \theta = 200 \sin \theta \]

\[ T = 200 \sin \theta \quad (1) \]
Problems (Equilibrium of Particles)

FBD of Particle 2

\[ \sum F_x = 0 \]
\[ -N_2 \sin(90 - \theta) + T \cos(90 - \theta) = 0 \]
\[ N_2 = \frac{T \cos(90 - \theta)}{\sin(90 - \theta)} \]

\[ \sum F_y = 0 \]
\[ N_2 \cos(90 - \theta) + T \sin(90 - \theta) - 150 = 0 \]
\[ \left( \frac{T \cos(90 - \theta)}{\sin(90 - \theta)} \right) \cos(90 - \theta) + T \sin(90 - \theta) - 150 = 0 \]
\[ T \cos^2(90 - \theta) + T \sin^2(90 - \theta) = 150 \sin(90 - \theta) \]

\[ T = 150 \sin(90 - \theta) \]  \( \text{(2)} \)

\[ (1)=(2) \quad 200 \sin \theta = 150 \sin(90 - \theta) \]

\[ \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{150}{200} \]

\[ \theta = 36.87^\circ \]

\[ T = 120 \text{ N}, \quad N_1 = 160 \text{ N}, \quad N_2 = 90 \text{ N} \]
13. Smooth collars $A$, $B$ and $C$, each weighing 360 N, are connected by the wires $AB$ and $BC$ and may slide freely on the smooth rod having the shape shown. Determine the magnitude of the horizontal force $P$ which must be applied to the collar $A$ to maintain equilibrium. $DEFG$ portion of the rod is parallel to $xy$-plane.
**FBD of collar A**

- **\( \vec{W} \)**
- **\( \vec{P} \)**
- **\( \vec{T}_{AB} \)**
- **\( \vec{N}_{Ax} \)**

**Four unknowns**

**FBD of collar B**

- **\( \vec{W} \)**
- **\( \vec{T}_{BA} \)**
- **\( \vec{T}_{BC} \)**
- **\( \vec{N}_{Bz} \)**
- **\( \vec{N}_{Bxy} \)**

**Four unknowns**

**FBD of collar C**

- **\( \vec{T}_{CB} \)**
- **\( \vec{N}_{Cx} \)**
- **\( \vec{N}_{Cz} \)**

**Three unknowns**
\[ \sum \vec{F} = 0 \quad \Rightarrow \quad T_{BC} + \vec{W} + \vec{N}_C = 0 \]

\[ \vec{T}_{BC} = T_{BC} \frac{-4\hat{i} + 4\hat{j} + 7\hat{k}}{9} \]

\[ \vec{W} = -360\hat{j} \]

\[ \vec{N}_C = N_{Cx}\hat{i} + N_{Cz}\hat{k} \]

\[ \sum F_y = 0 \quad \Rightarrow \quad \frac{4}{9} T_{BC} - 360 = 0 \]

\[ T_{BC} = 810 \; \text{N} \]
\[ \sum \vec{F} = 0 \quad \Rightarrow \quad \vec{T}_{BC} + \vec{T}_{AB} + \vec{W} + \vec{N}_B = 0 \]

\[ \vec{T}_{BC} = -\vec{T}_{CB} = 360\hat{i} - 360\hat{j} - 630\hat{k} \]

\[ \vec{T}_{BA} = \frac{T_{BA}}{6} \left( -2\hat{i} + 4\hat{j} - 4\hat{k} \right) \]

\[ \vec{W} = -360\hat{j} \]

\[ \vec{N}_B = 0.6N_{Bxy}\hat{i} + 0.8N_{Bxy}\hat{j} + N_{Bz}\hat{k} \]

\[ \sum F_x = 0 \quad \Rightarrow \quad 0.6N_{Bxy} + 360 - \frac{2}{6}T_{BA} = 0 \]

\[ \sum F_y = 0 \quad \Rightarrow \quad 0.8N_{Bxy} - 360 + \frac{4}{6}T_{BA} - 360 = 0 \]

\[ T_{BA} = 1080 \text{ N} \]

\[ \theta = 36.87^\circ \]

\[ \sin \theta = 0.6 \]

\[ \cos \theta = 0.6 \]
\[ \sum \vec{F} = 0 \quad \Rightarrow \quad \vec{T}_{AB} + \vec{W} + \vec{N}_A + \vec{P} = 0 \]

\[ \vec{T}_{AB} = -\vec{T}_{BA} = 360\hat{i} - 720\hat{j} + 720\hat{k} \]

\[ \vec{W} = -360\hat{j} \]

\[ \vec{N}_A = N_{Ax}\hat{i} + N_{Ay}\hat{j} \]

\[ \vec{P} = -P\hat{k} \]

\[ \sum F_z = 0 \quad \Rightarrow \quad 720 - P = 0 \]

\[ P = 720 \text{ N} \]
14. Smooth collars $A$ and $B$ are connected by the spring. Spring has a constant of 120 N/cm and its unstretched length is 30 cm. Determine the magnitude of the force $P$ which must be applied to the collar $A$ to maintain equilibrium and the reaction between the collar and bar. Neglect the weight of the collars. Take $A (40;0;40)$ and $B (0;20;80)$. 

![Diagram of the system with collars A and B connected by a spring, showing the forces P and Q, and the reaction between the collar and bar. The diagram includes coordinates and distances relevant to the problem.]
FBD of collar A

\[ \sum \vec{F} = 0 \quad \Rightarrow \quad \vec{F}_{spring} + \vec{P} + \vec{N}_A = 0 \]

\[ \vec{F}_{spring} = 120(60 - 30) \frac{-40\hat{i} + 20\hat{j} + 40\hat{k}}{60} \]

\[ \vec{F}_{spring} = -2400\hat{i} + 1200\hat{j} + 2400\hat{k} \]

\[ \vec{AB} = (0 - 40)\hat{i} + (20 - 0)\hat{j} + (80 - 40)\hat{k} \]

\[ \vec{AB} = -40\hat{i} + 20\hat{j} + 40\hat{k} \quad \Rightarrow \quad |\vec{AB}| = 60 \text{ cm} \]

\[ \vec{P} = \frac{3}{5} \vec{P} - \frac{4}{5} \vec{P}_k \]

\[ \vec{N}_A = \frac{4}{5} N_{Axz}\hat{i} + \frac{3}{5} N_{Axz}\hat{k} + N_{Ay}\hat{j} \]

\[ \sum F_x = 0 \quad \Rightarrow \quad -2400 + \frac{3}{5} P + \frac{4}{5} N_{Axz} = 0 \]

\[ \sum F_y = 0 \quad \Rightarrow \quad 1200 + N_{Ay} = 0 \quad \Rightarrow \quad N_{Ay} = -1200 \text{ N} \]

\[ \sum F_z = 0 \quad \Rightarrow \quad 2400 - \frac{4}{5} P + \frac{3}{5} N_{Axz} = 0 \]

\[ N_{Axz} = 387 \text{ N} \quad P = 3483.87 \text{ N} \]

Correct sense
15. The 10-kg collar A and 20-kg collar B rest on the smooth fixed bars by the cable AB and the force $F$. Determine the magnitude of the force $F$ required to maintain equilibrium of the system. The force $F$ is parallel to the bar $CE$.

$AB = 3.5 \ m$
16. Collar A weighing 170 N, which can slide freely on the quarter circle, is held in equilibrium by cable AB.  

a) Determine the tension in the cable.  

b) Determine the magnitude and the components of the contact force acting on the collar from the circle.