7.1 Piezoelectric sensors: (Silva p.253)

Piezoelectric materials such as lead-zirconate-titanate (PZT) can generate electrical charge and potential difference when they are subjected to mechanical stress or strain. This piezoelectric is used in piezoelectric transducers. It is also noted that reverse piezoelectric effect means that piezoelectric materials can serve as actuators when they are subjected to a potential difference (charge or electrical field).

- Sensors (piezoelectric): Accelerometer, velocity, torque, force.
- Actuators (reverse piezoelectric): Valves, motors, micromotion

Piezoelectric elements have anisotropic material properties. When an electric field is applied to the material, a polarization occurs in the material. The direction of the polarization is important.
Piezoelectric disc or plate with two electrodes on the opposite faces can be modeled with capacitance $C$ as a capacitor. The equivalent circuit representation of a piezoelectric sensor with a charge source and a capacitor for a quartz crystal is shown in the figure.

![Equivalent circuit of a piezoelectric sensor](image)

The sensitivity of a piezoelectric crystal may be represented either by its charge sensitivity or by its voltage sensitivity.

**Charge sensitivity:**
$$S_q = \frac{\text{generated charge}}{\text{applied force}}$$

**Voltage sensitivity:**
$$S_v = \frac{\text{generated voltage}}{\text{applied pressure (or stress)}}$$

$$S_q = kS_v$$

$k$: dielectric constant (permittivity)
A barium titanate crystal has a charge sensitivity of 150.0 picocoulombs per newton (pC/N). (Note: 1 pC = $1 \times 10^{-12}$ coulombs; coulombs = farads x volts). The dielectric constant for the crystal is $1.25 \times 10^{-8}$ farads per meter (F/m). From Equation 4.63, the voltage sensitivity of the crystal is computed as

$$S_v = \frac{150.0 \text{ pC/N}}{1.25 \times 10^{-8} \text{ F/m}} = \frac{150.0 \times 10^{-12} \text{ C/N}}{1.25 \times 10^{-8} \text{ F/m}} = 12.0 \times 10^{-3} \text{ V.m/N} = 12.0 \text{ mV.m/N}.$$ 

<table>
<thead>
<tr>
<th>Material</th>
<th>Charge Sensitivity $S_q$ (pC/N)</th>
<th>Voltage Sensitivity $S_v$ (mV.m/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead Zirconate Titanate (PZT)</td>
<td>110</td>
<td>10</td>
</tr>
<tr>
<td>Barium Titanate</td>
<td>140</td>
<td>6</td>
</tr>
<tr>
<td>Quartz</td>
<td>2.5</td>
<td>50</td>
</tr>
<tr>
<td>Rochelle Salt</td>
<td>275</td>
<td>90</td>
</tr>
</tbody>
</table>

The sensitivity of a piezoelectric element is dependent on the direction of loading.
7.2 Accelerometers:

Acceleration is the measure of the change in the velocity of an object with respect to time.

\[ a = \frac{dv}{dt} \]

SI unit: m/s²

1 g = 9.81 m/s²

Newton’s second law: A force (f) is necessary to accelerate a mass, and its magnitude is given by the product (m) and acceleration (a).

\[ f = ma \]

D’Alembert’s principle: If a force of magnitude m*a were applied to the accelerating mass in the direction opposing the acceleration, then the system can be analyzed using static considerations.

Accordingly, mass can serve as front-end element to convert acceleration into force.
Accelerometers: Change in Acceleration as a change in Voltage.

Most commonly used accelerometer types:
• Piezoelectric accelerometers.
• Strain based accelerometers.

7.2.1 Piezoelectric Accelerometers:

A piezoelectric element measures the inertia force caused by acceleration.

Advantages:
• Light weight
• High frequency response (up to 1 MHz.)
• High fundamental natural freq. (typically 20 kHz)
• Useful or operating range is typically up to 1Hz-5 kHz

Disadvantages:
• High output impedance
• Low output voltage (need charge amplifier)
Typical accelerometer sensitivities are 10 pC/g and 5 mV/g and sensitivity depends on piezoelectric properties.

Common mounting techniques:
1. Screw in base (typically up to 5 kHz)
2. Glue, cement or wax (typically up to 5 kHz)
3. Magnetic base (typically up to 3 kHz)
4. Spring-base mount (typically up to 500 Hz)
5. Hand-held probe (typically up to 500 Hz)

7.2.2 Charge Amplifier:

Piezoelectric accelerometers requires using charge amplifiers due to the reasons:
1. High output impedance and small output signal in the sensor results.
2. The charge can leak out through the load.

\[ R_f C_f \frac{dv_0}{dt} + v_0 = -R_f \frac{dq}{dt} \]

- \( R_f \): Feedback resistance of amplifier
- \( C_f \): Feedback capacitance of amplifier
- \( K \): Open-loop gain
- \( C \): Sensor capacitance
- \( C_c \): Cable capacitance
- \( q \): Charge (Coulomb)
- \( v_0 \): Sensor output voltage (Volt)
Charge amplifiers equations:

\[ R_f C_f \frac{dv_o}{dt} + v_o = -R_f \frac{dq}{dt} \]

Charge amplifier’s differential equation

\[ \frac{v_o(s)}{q(s)} = -\frac{R_f s}{[R_f C_f s + 1]} \]

Transfer function

\[ G(j\omega) = \frac{j\tau_c \omega}{[j\tau_c \omega + 1]} \]

Frequency transfer function

\[ M = \frac{\tau_c \omega}{\sqrt{\tau_c^2 \omega^2 + 1}} \]

Magnitude of the frequency transfer function

As \( \omega \to \infty \), note that \( M \to 1 \).

Measurement accuracy depends on the closeness of \( M \) to 1.
Piezoelectric accelerometer output voltage: 

\[ V_0 = -\frac{q}{C_f} \left( 1 + \frac{1}{KC_f} (C + C_f + C_c) \right) \]

\[ V_0 = -\frac{q}{C_f} \quad K >> \]

\[ \tau_c = R_f C_f \]

- Time constant affect charge amplifier's output. When considering the time constant, the user must think in terms of either frequency or time domain.
- The longer the time constant, the better the low-end frequency response and the longer the usable measuring time.

\[ R_f: \text{Feedback resistance of amplifier} \]
\[ C_f: \text{Feedback capacitance of amplifier} \]
\[ K: \text{Open-loop gain} \]
\[ C: \text{Sensor capacitance} \]
\[ C_c: \text{Cable capacitance} \]
\[ q: \text{Charge (Coulomb)} \]
\[ v_0: \text{Sensor output voltage (Volt)} \]
Example-7.1: (Silva p.279)

A schematic diagram of a strain-gage accelerometer is shown in the figure. A point mass is used as the acceleration sensing element at the tip cantilever beam with a strain-gage. The following parameters and numerical values are given below.

a) Find the maximum acceleration that could be measured using the accelerometer.
b) Determine the sensitivity of the accelerometer in microV/gm
c) How much amplification would be needed so that the maximum acceleration corresponds to the upper limit of ADC (10 V)?

![Image of accelerometer schematic](image)

- $M = 5 \text{ gr}$
- $E = 5 \times 10^{10} \text{ N/m}^2$
- $L = 1 \text{ cm}$
- $b = 1 \text{ mm}$
- $h = 0.5 \text{ mm}$
- $S_s (GF) = 200$
- $v_{ref} = 20 \text{ V}$

$W = Mg =$ weight of the seismic mass at the free end of the cantilever element

$E =$ Young's modulus of the cantilever

$\ell =$ length of the cantilever

$b =$ cross-section width of the cantilever

$h =$ cross-section height of the cantilever

$S_s =$ gage factor (sensitivity) of each strain-gage

$v_{ref} =$ supply voltage to the bridge.
7.3 Laser Displacement Sensors: (Keyence manual)

Main features:

<table>
<thead>
<tr>
<th>Feature</th>
<th>LK-G157</th>
<th>LK-G37</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Accuracy</td>
<td>Small spot</td>
<td>LK-G32</td>
</tr>
<tr>
<td>Wide beam</td>
<td>LK-G37</td>
<td></td>
</tr>
<tr>
<td>Long Distance</td>
<td>Small spot</td>
<td>LK-G152</td>
</tr>
<tr>
<td>Wide beam</td>
<td>LK-G157</td>
<td></td>
</tr>
</tbody>
</table>

Some engineering measurement applications:

- Thickness
- Height
- Vibration
Acceleration and Displacement Measurement