KINETICS OF PARTICLES

WORK AND ENERGY
**WORK**: Figure shows a force $\vec{F}$ acting on a particle at A which moves along the path. The position vector $\vec{r}$ measured from some convenient origin O locates the particle as it passes point A, and $d\vec{r}$ is the differential displacement from A to A'. The work done by the force during the displacement is defined as

$$dU = \vec{F} \cdot d\vec{r}$$
The magnitude of this dot product is \( dU = Fd\cos \alpha \), where \( ds \) is the magnitude of \( d\vec{r} \). This expression may be interpreted as the displacement multiplied by the force component \( F_t = F \cos \alpha \) in the direction of the displacement.

With this definition of work, it should be noted that the component \( F_n = F \sin \alpha \) normal to the displacement does no work.

The SI units of work are those of force (N) times displacement (m) of N·m. This unit is given joule (J).
Calculation of Work

During a finite movement of the point of application of a force, the force does an amount of work equal to

\[ U = \int \vec{F} \cdot d\vec{r} = \int \left( F_x \, dx + F_y \, dy + F_z \, dz \right) \]

or

\[ U = \int F_t \, ds \]

In order to carry out this integration, it is necessary to know the relations between the force components and their respective coordinates or the relation between \( F_t \) and \( s \).
We now consider the work done on a particle of mass \( m \) moving along a curved path under the action of the force \( F \), which is the resultant of all forces acting on the particle.

The work done by \( F \) during a finite movement of the particle from point 1 to point 2 is

\[
U_{1-2} = \int_1^2 \overrightarrow{F} \cdot d\overrightarrow{r} = \int_1^2 F_t \, ds
\]

When we substitute Newton’s second law, the expression becomes

\[
U_{1-2} = \int_1^2 \overrightarrow{F} \cdot d\overrightarrow{r} = \int_1^2 m\overrightarrow{a} \cdot d\overrightarrow{r}
\]
where $\mathbf{a}_t$ is the tangential component of the acceleration. In terms of the velocity $\mathbf{v}_t \, ds = \mathbf{v} \, dv$. Thus the expression for work becomes

$$U_{1-2} = \int_{v_1}^{v_2} \mathbf{F} \cdot d\mathbf{r} = \int_{v_1}^{v_2} m \mathbf{v} \, dv = \frac{1}{2} m \left( v_2^2 - v_1^2 \right)$$

where $\mathbf{F} = \Sigma \mathbf{F}$.
The kinetic energy $T$ of the particle is defined as

$$T = \frac{1}{2}mv^2$$

and is the total work which must be done on the particle to bring it from a state of rest to a velocity $v$. Kinetic energy $T$ is a scalar quantity with the units of “Nm” or “joule (J)” in SI units. Kinetic energy is always positive, regardless of the direction of the velocity.

Expression for the work may be written as

$$U_{1-2} = T_2 - T_1 = \Delta T$$

which is the work-energy equation of particle. The equation states that the total work done by all forces acting on a particle as it moves from point 1 to point 2 equals the corresponding change in kinetic energy of the particle. Although $T$ is always positive, the change $\Delta T$ may be negative, positive or zero.
We consider first the motion of a particle of mass $m$ in close proximity to the surface of the earth, where the gravitational attraction (weight) $mg$ is essentially constant. The gravitational potential energy $V_g$ of the particle is defined as

$$V_g = mgh$$

The change in potential energy is

$$\Delta V_g = mg(h_2 - h_1) = mg\Delta h$$
The second example of potential energy occurs in the deformation of an elastic body, such as a spring. Elastic potential energy is

\[ V_e = \int_{0}^{x} kx \, dx = \frac{1}{2} kx^2 \]

The change in elastic potential energy is

\[ \Delta V_e = \frac{1}{2} k \left( x_2^2 - x_1^2 \right) \]
With the elastic member included in the system, we may write the work-energy equation as

\[ U_{1-2} = \Delta T + \Delta V_g + \Delta V_e \]

or

\[ T_1 + V_{g1} + V_{e1} + U_{1-2} = T_2 + V_{g2} + V_{e2} \]

We may rewrite the alternative work-energy equation as

\[ U_{1-2} = \Delta \left( T + V_g + V_e \right) \]

Where \( E=T+V_g+V_e \) is the total mechanical energy of the particle. For problems where \( U_{1-2} \) term is zero, and the energy equation becomes

\[ \Delta E = 0 \quad \text{or} \quad E=\text{constant} \]

This equation expresses the law of conservation of dynamical energy.
The capacity of a machine is measured by the time rate at which it can do work or deliver energy. The total work or energy output is not a measure of this capacity since a motor, no matter how small, can deliver a large amount of energy if given sufficient time. Thus, the capacity of a machine is rated by its **power**, which is defined as the time rate of doing work.
When a force $\vec{F}$ does work an amount $U$, the power $P$ will be

$$P = \frac{dU}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \frac{d\vec{r}}{dt} = \vec{v}$$

$$P = \vec{F} \cdot \vec{v}$$

or

$$P = \vec{M} \cdot \vec{\omega}$$

Power is a scalar quantity.

In SI system it has the units of $\text{N} \cdot \text{m/s}=\text{Joule/s}=\text{Watt} \ (W)$. 
1. The 7 kg collar $A$ slides with negligible friction on the fixed vertical shaft. When the collar is released from rest at the bottom position shown, it moves up the shaft under the action of the constant force $F=200$ N applied to the cable. Calculate the stiffness $k$ which the spring must have if its maximum compression is to limited to 75 mm. The position of the small pulley at $B$ is fixed.
2. The 2 kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.4. Calculate (a) the velocity $v$ of the collar as it strikes the spring and (b) the maximum deflection $x$ of the spring.
3. Small metal blocks are discharged with a velocity of 0.45 m/s to a ramp by the upper conveyor shown. If the coefficient of kinetic friction between the blocks and the ramp is 0.30, calculate the angle $\theta$ which the ramp must make with the horizontal so that the blocks will transfer without slipping to the lower conveyor moving at the speed of 0.15 m/s.
4. The 1.2 kg slider is released from rest in position A and slides without friction along the vertical-plane guide shown. Determine (a) the speed $v_B$ of the slider as it passes position B and (b) the maximum deflection $\delta$ of the spring.
5. The light rod is pivoted at $O$ and carries the 2- and 4-kg particles. If the rod is released from rest at $\theta=60^\circ$ and swings in the vertical plane, calculate (a) the velocity $v$ of the 2 kg particle just before it hits the spring in the dashed position and (b) the maximum compression $x$ of the spring. Assume that $x$ is small so that the position of the rod when the spring is compressed is essentially horizontal.
6. The mechanism is released from rest with $\theta = 180^\circ$, where the uncompressed spring of stiffness $k = 900 \text{ N/m}$ is just touching the underside of the 4-kg collar. Determine the angle $\theta$ corresponding to the maximum compression of the spring. Motion is in the vertical plane, and the mass of the links may be neglected.
PROBLEMS

7. The 0.6-kg slider is released from rest at A and slides down the smooth parabolic guide (which lies in a vertical plane) under the influence of its own weight and of the spring of constant 120 N/m. Determine the speed of the slider as it passes point B and the corresponding normal force exerted on it by the guide. The unstretched length of the spring is 200 mm.
8. The system shown is in equilibrium when $\phi = 0^\circ$. Initially when block C is in a state of rest at $\phi = 90^\circ$, it is given a slight push. Determine the velocity of the block as it passes from the position where $\phi = 37^\circ$. Neglect the mass of the light rod.
PROBLEMS

9. A thin circular rod is supported in a *vertical* plane by a bracket at A. Attached to the bracket and loosely wound around the rod is a spring of constant \(k=45\) N/m and undeformed length equal to the arc of circle AB. A 220 gram collar C, not attached to the spring, can slide without friction along the rod. Knowing that the collar is released from the rest when \(\theta=30^\circ\), determine (a) the maximum height above point B reached by the collar, (b) the maximum speed of the collar.
10. The 25 kg slider in the position shown has an initial velocity 0.6 m/s on the inclined rail and slides under the influence of gravity and friction. The coefficient of kinetic friction between the slider and rail is 0.5. Calculate the velocity of the slider as it passes the position for which the spring is compressed a distance $x=100$ mm.