CONSTRAINED MOTION OF CONNECTED PARTICLES
Sometimes the motions of particles are interrelated because of the constraints imposed by interconnecting members. In such cases, it is necessary to account for these constraints in order to determine the respective motions of the particles.

**One Degree of Freedom:**
Consider first the very simple system of two interconnected particles A and B. It should be quite evident by inspection that the horizontal motion of A is twice the motion of B. We can illustrate this by using the method of analysis which applies to more complex situations where the results cannot be easily obtained by inspection.

The total length of the cable is

\[ L = x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b \]

(Here, \( L, r_2, r_1, \pi, b \) are constant)
\[ L = x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b \]

The first and second time derivatives of the equation give:

\[ 0 = \dot{x} + 2\dot{y} \quad \text{or} \quad 0 = v_A + 2v_B \]

\[ 0 = \ddot{x} + 2\ddot{y} \quad \text{or} \quad 0 = a_A + 2a_B \]

The velocity and acceleration constraint equations indicate that, for the coordinates selected, the velocity of \( A \) must have a sign which is opposite to that of the velocity of \( B \), and similarly for the accelerations. The constraint equations are valid for motion of the system in either direction. We emphasize that \( v_A \) is positive to the left and that \( v_B \) is positive down.
The system with two degrees of freedom is shown here. The positions of the lower cylinder and pulley C depend on the separate specifications of the two coordinates $y_A$ and $y_B$.

1. **Cable Length:**

$$L_1 = y_A + 2y_D + \text{const} \tan t$$

$$0 = \dot{y}_A + 2\dot{y}_D \quad \Rightarrow \quad 0 = \ddot{y}_A + 2\ddot{y}_D$$

2. **Cable Length:**

$$L_2 = y_B + y_C + (y_C - y_D) + \text{const} \tan t$$

$$0 = \dot{y}_B + 2\dot{y}_C - \dot{y}_D \quad \Rightarrow \quad 0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$$

$$0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$$

or

$$0 = v_B + 2v_C - v_D$$

$$0 = a_B + 2a_C - a_D$$
PROBLEMS

1. Cylinder B has a downward velocity of 0.6 m/s and upward acceleration of 0.15 m/s². Calculate the velocity and acceleration of block A.

Cable Length:

\[ L = 2s_A + 3s_B + \text{constant} \]

\[ 0 = 2v_A + 3v_B \quad \Rightarrow \quad 0 = 2a_A + 3a_B \]

\[ v_A = -1.5v_B = -1.5(0.6) = -0.9 \text{ m/s (Up the incline) } \]

\[ a_A = -1.5a_B = -1.5(-0.15) = 0.225 \text{ m/s}^2 \text{ (Down the incline) } \]
2. For the pulley system shown, each of the cables at A and B is given a velocity of 2 m/s in the direction of the arrow. Determine the upward velocity $v$ of the load $m$.

\[
x_1 + 2y_1 = const \tan t \quad \Rightarrow \quad \dot{x}_1 + 2\dot{y}_1 = 0 \quad \dot{y}_1 = -\frac{\dot{x}_1}{2}
\]

\[
x_2 + y_2 + (y_2 - y_1) = const \tan t \quad \Rightarrow \quad \dot{x}_2 + 2\dot{y}_2 - \dot{y}_1 = 0
\]

\[
\dot{x}_2 + 2\dot{y}_2 - \left(-\frac{\dot{x}_1}{2}\right) = 0
\]

\[
\dot{y}_2 = -\frac{\dot{x}_2}{2} - \frac{\dot{x}_1}{4} = -\frac{2}{2} - \frac{2}{4} = -1.5 \text{ m/s}
\]
PROBLEMS

3. Neglect the diameters of the small pulleys and establish the relationship between the velocity of \( A \) and the velocity of \( B \) for a given value of \( y \).

\[
L = 2x + 3\sqrt{y^2 + b^2} + \text{const} \tan t
\]

\[
\dot{L} = 2\dot{x} + 3\frac{yy\dot{y}}{\sqrt{y^2 + b^2}} = 0 \quad \dot{x} = v_A \quad \text{and} \quad \dot{y} = v_B
\]

\[
v_B = -\frac{3yyA}{2\sqrt{y^2 + b^2}}
\]
4. Collars \( A \) and \( B \) slide along the fixed right-angle rods and are connected by a cord of length \( L \). Determine the acceleration of collar \( B \) as a function of \( y \) if collar \( A \) is given a constant upward velocity \( v_A \).

\[
x^2 + y^2 = L^2 \quad \Rightarrow \quad x^2 = L^2 - y^2
\]

\[
2x\ddot{x} + 2y\ddot{y} = 0 \quad \Rightarrow \quad \ddot{x} = -\frac{y\ddot{y}}{x}
\]

\[
\ddot{x}^2 + x\dddot{x} + \dot{y}^2 + y\dddot{y} = 0 \quad \ddot{y} = v_A = \text{constant} \quad \ddot{y} = 0
\]

\[
\ddot{x} = -\frac{\dot{x}^2 + \dot{y}^2}{x} = -\left( -\frac{y\ddot{y}}{x} \right)^2 + \dot{y}^2
\]

\[
= -\frac{L^2 v_A^2}{(L^2 - y^2)^{3/2}}
\]